

Table 2.6 Commonly used daughter distribution functions.

Name	$b(L/\lambda)$
Symmetric fragmentation	$2\delta\left(L - \frac{\lambda}{2^{1/3}}\right)$
Parabolic distribution	$\frac{3CL^2}{\lambda^3} + \left(1 - \frac{C}{2}\right)\left(\frac{72L^8}{\lambda^9} - \frac{72L^5}{\lambda^6} + \frac{18L^2}{\lambda^3}\right)$
Mass ratio 1:2	$\delta\left(L - \lambda\left(\frac{1}{3}\right)^{1/3}\right) + \delta\left(L - \lambda\left(\frac{2}{3}\right)^{1/3}\right)$
Uniform distribution	$2/\lambda$
Attrition	$2\delta(L - \lambda)$

conserved in the breakage process. There are two limiting daughter distribution functions for binary breakage: the symmetric fragmentation, when two, equal sized crystals are produced, and attrition, when an infinitesimally small crystal is produced, and the other has the size of parent crystal. This kernel is often used to model the collision induced secondary nucleation. It is worth noticing that other distribution functions were also developed and successfully applied to crystallization processes, which are summarized in Table 2.6.¹⁵

The death rate of λ sized crystals by breakage, as a function of its number n_λ and the selection function $S(\lambda)$, can be calculated as:

$$D_{\text{bre}} = S(\lambda)n_\lambda \quad (2.18)$$

According to eqn (2.18), the breakage, in contrast with agglomeration, is a first order process, showing linear dependency on the particle number. Then, the formation of crystals of size L from breakage of crystals of size λ can be obtained, in the knowledge of the daughter distribution function:

$$B_{\text{bre}} = b(L/\lambda)S(\lambda)n_\lambda \quad (2.19)$$

Naturally, the breakage description can be extended to population of crystals $n(L,t)$:

$$\frac{\partial n(L,t)}{\partial t} = B_{\text{bre}}(L) - D_{\text{bre}}(L) \quad (2.20)$$

With the terms:

$$D_{\text{bre}}(L) = S(L)n(L,t) \quad (2.21)$$

$$B_{\text{bre}}(L) = \int_{\lambda}^{L_{\text{max}}} b(L/\lambda)n(\lambda,t)S(\lambda)d\lambda \quad (2.22)$$

$B_{\text{bre}}(L)$ function gives the number of crystals of size L produced from the breakage of a $\lambda > L$ crystal, which depends on the rate of breakage of λ crystals