$$U_m = U_{cS} + U_{dS} = U_{LS} + U_{GS} \tag{5.1}$$

Reynolds, Weber, and Eötvös numbers are defined by

$$\mathrm{Re}_L = \frac{DU_m}{\nu_L}, \tag{5.2}$$

$$\mathrm{We}_L = \frac{\rho_L D U_m{}^2}{\sigma}, \tag{5.3}$$

$$\mathrm{Eo}_D = \frac{D^2\left(\rho_L - \rho_G\right) g \cos\beta'}{8\sigma}, \tag{5.4}$$

respectively. In the latter equation, β' is defined by

$$\beta' = \begin{Bmatrix} |\beta|; |\beta| < \pi/4 \\ \pi/2 - |\beta|; |\beta| > \pi/4 \end{Bmatrix} \tag{5.5}$$

where β is the inclination of the tube from the horizontal.

The gas-phase holdup is

$$\varepsilon_G = \frac{Q_G}{Q_G + Q_L}, \tag{5.6}$$

where Q_G and Q_L denote the volumetric flow rates of gas and liquid, respectively, through the tube.

5.2.3 Flow Transition of Slug Flow

The flow regime map shown in Figure 5.5 shows several transition boundaries. To maintain stable slug-flow conditions, the most important boundaries are those surrounding the slug-flow region. Thus only the envelope around the stable slug-flow region is discussed here. Information on the other transitions in Figure 5.5 is available in Ullmann and Brauner.[15]

5.2.3.1 Transition from Bubbly to Slug-flow Regime

Starting from the left side of the slug-flow region, the transition from bubbly to slug flow should involve surface tension through the Eötvös number, which is small for the corresponding crystallization in micro- and minitubes. Under this condition, Ullmann and Brauner suggest the relationship,

$$U_{LS} = \frac{1 - \left(\varepsilon_G\right)_{crit}}{\left(\varepsilon_G\right)_{crit}} U_{GS}, \tag{5.7}$$

where $(\varepsilon_G)_{crit}$ is 0.15. Eqn (5.7) is plotted in Figure 5.5 as the bubbly flow to slug-flow transition line B-S.