where D ($m^2\ s^{-1}$) is the axial diffusivity. However, it has been found that axial dispersion is often not sufficient to describe the experimentally observed broadening of the product CSD.[22] In such a case, a dispersion term that takes into account the growth rate dispersion can be incorporated as follows.

$$\frac{\partial}{\partial t}(n)+\frac{\partial}{\partial L}(Gn)+\frac{\partial}{\partial x}(un)=D_{\mathrm{G}}\frac{\partial^2}{\partial L^2}(n) \tag{2.39}$$

where D_{G} ($\mu m^2\ s^{-1}$) is growth diffusivity for the random fluctuations of the growth rate of the crystals, which is discussed in the last section of this chapter.

This PBE needs to be supplemented with appropriate mass and energy balance equations. Assuming that no crystallizer fouling or encrustation takes place and crystal volume is negligible compared to the liquid volume, the mass balance equation can be written as:

$$\frac{\partial}{\partial t}(c)+\frac{\partial}{\partial x}(uc)=-\rho_{\mathrm{c}}k_{\mathrm{V}}\frac{\partial}{\partial t}(\mu_3) \tag{2.40}$$

Where k_{V} (−) is the volume shape factor for the crystals, ρ_{c} ($kg\ m^{-3}$) is the crystal density, and μ_3 (μm^3) is the third moment of the CSD which can be considered as a measure of crystal volume. The derivation of the population balance, mass balance and energy balance equations can be found in the Appendices. In eqn (2.40), the first term denotes the rate of change in concentration along the PFC, the second term is the advection term which takes into account the change in concentration due to fluid flow and the term on the right hand side is the depletion term due to crystal growth.

2.4.1 Case Study: PFC With Multiple Feeding Points

In this section, we present a case study on antisolvent crystallization in a tubular crystallizer. Antisolvent is a fluid which, when mixed with the solution of an active pharmaceutical ingredient (API), reduces the solubility of the API in the mixed solvent. Antisolvent crystallization is particularly important for separation of substances which are temperature sensitive. The system considered here is the crystallization of flufenamic acid in ethanol (solvent) and water (antisolvent). The case study is based on the work by Alvarez and Myerson.[22] The PFC consists of four glass reactor modules each of which has a volume of 76 cm^3 (600 mm long, 12.7 mm internal diameter). The crystallizer is maintained at 25°C using a water bath temperature control. The feed solution contains 1.4 mg mL^{-1} of flufenamic acid in ethanol and the feed rate is maintained at 100 $mL\ min^{-1}$. One of the objectives of this study is to investigate the effect of number of injection points of the antisolvent on the CSD. Thus, the total antisolvent flowrate 200 $mL\ min^{-1}$ is distributed equally in 1,2,3 and 4 injection points, *e.g.*, one flow of 200 $mL\ min^{-1}$ for one injection point,