$$A\Delta x\Delta L\left(n|_{t+\Delta t} - n|_t\right) = A\Delta L\Delta t\left(un|_x - un|_{x+\Delta x}\right)$$
$$+A\Delta x\Delta t\left(G^{(k)}n|_L - G^{(k)}n|_{L+\Delta L}\right) + SA\Delta x\Delta L\Delta t$$

If the cross-sectional area of the PFC is constant, then we have

$$\Rightarrow \frac{n|_{t+\Delta t} - n|_t}{\Delta t} = \frac{un|_x - un|_{x+\Delta x}}{\Delta x} + \frac{Gn|_L - Gn|_{L+\Delta L}}{\Delta L} + S$$

In the limit Δt, $\Delta x \rightarrow 0$, we have

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{\partial}{\partial L}(Gn) - \frac{\partial}{\partial x}(un) + S \tag{A1}$$

The source term S due to aggregation and breakage can be set to be zero if these processes are negligible. Then we have

$$\Rightarrow \frac{\partial}{\partial t}(n) + \frac{\partial}{\partial L}(Gn) + \frac{\partial}{\partial x}(un) = 0 \tag{A2}$$

The boundary and initial conditions are:

$$n(L_0, x, t) = \frac{B_0}{G},$$
$$n(L,0,t) = n_{\text{seed}}(L,t),$$
$$n(L,x,0) = n_0(L,x) \tag{A3}$$

A2 Derivation of the Mass Balance Equation for Plug Flow Crystallizer

Taking mass balance for the solute species present in the liquid phase in the volume element $A\Delta x$, as shown in Figure 2.18, we have,

$$\left(A\Delta x c\right)\Big|_t^{t+\Delta t} = -\left(\Delta t u A c\right)\Big|_x^{x+\Delta x}$$
$$-\Delta t A\Delta x \rho_c \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{L_0}^{L_{\max}} nL^3 \,\mathrm{d}L\right) - \Delta t A\Delta x k_V \rho_c B_0 L_0^3,$$

where on the right hand side of the equation, the contributing terms are due to convection by fluid flow, depletion due crystal growth and nucleation, respectively.

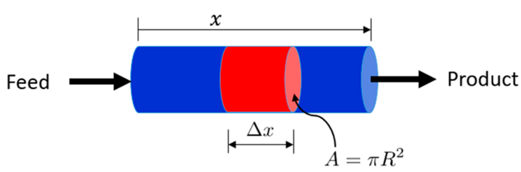

Figure 2.18 Schematic of a plug flow crystallizer showing volume element.