Table 2.9 Statistical properties of the particle population calculated from the moments of the distribution.

Property	Expression in terms of moments
Number based mean size	μ_1/μ_0
Sauter mean diameter	μ_3/μ_2
Volume mean diameter	μ_4/μ_3
Variance (of equivalent normal PDF)	$\sqrt{\frac{\mu_2}{\mu_0}-\left(\frac{\mu_1}{\mu_0}\right)^2}$
Coefficient of variation	$\sqrt{\frac{\mu_0\mu_2}{\mu_1^2}-1}$

In this case, the moment equation becomes:

$$\begin{aligned}\frac{\mathrm{d}\,\mu_k}{\mathrm{d}t} &= BL_n^k + \int_0^\infty kL^{k-1}GL^j n(L,t)\,\mathrm{d}L \\ +\frac{\mu_{f,k}-\mu_k}{\tau} &= BL_n^k + kG\mu_{k-1+j} + \frac{\mu_{f,k}-\mu_k}{\tau}\end{aligned} \tag{2.51}$$

It is obvious that the moment equations can be solved if and only if $j = 0$ or 1. However, in the applied side, no such restrictions exist for the value of size exponent. This is the so-called closure problem of the SMOM. The most generic solution to overcome the closure problem is the quadrature approximation of the population density function:

$$\tilde{n}(L,t) = \sum_{i=1}^{I} w_i(t)\,\delta\left[L - L_i(t)\right] \tag{2.52}$$

where i denotes the number of points used in the approximation, w_i is a quadrature weight and L_i is a quadrature abscissa. Applying the quadrature approximation eqn (2.52) to the moment transformation eqn (2.46), the moments can be approximated in terms of quadrature weights and abscissas:

$$\mu_k = \int_0^\infty L^k n(L,t)\,\mathrm{d}L \cong \int_0^\infty L^k \sum_{i=1}^{I} w_i(t)\,\delta\left[L - L_i(t)\right]\mathrm{d}L = \sum_{i=1}^{I} w_i(t)\,L_i^k(t) \tag{2.53}$$

Using the quadrature approximation, the system of moment equations can be approximated, and this is known as the quadrature method of moments (QMOM).[27] Then, using the simplified notations for better readability $w_i(t) \rightarrow w_i$ and $L_i(t) \rightarrow L_i$, eqn (2.51) becomes:

$$\frac{\mathrm{d}\,\mu_k}{\mathrm{d}t} \cong BL_n^k + kG\sum_{i=1}^{I} w_i L_i^{k-1+j} + \frac{1}{\tau}\left(\sum_{i=1}^{I} w_{f,i} L_{f,i}^k - \sum_{i=1}^{I} w_i L_i^k\right) \tag{2.54}$$

According to eqn (2.54) it is obvious that the closure problem doesn't exist anymore in the QMOM. Various QMOM implementations were developed[28]