cylinder), compared to a concave shape in hydrophilic tubing materials (*e.g.*, quartz tubing) with high affinity of solvent water to the material of the inner tubing wall. In flow systems with wall dragging the liquid, the front of a moving slug shows different curvature from the concave back of the slurry slug with respect to flow direction (Figures 5.3a, b and d).

5.3.2 Flow Analysis for Recirculation within Slugs

5.3.2.1 Dimensionless Recirculation Time

The rapid internal circulation of liquid slugs in capillary flow benefits crystallization by enhanced mixing as well as limiting adherence of crystals to the wall.[2] This internal circulation has been investigated for more than 50 years.[24–27] Taylor[24] carried out some simple experiments and suggested some possible flow patterns. Later Thulasidas *et al.*[25] used particle imaging velocimetry (PIV) to visualize recirculation patterns with a high degree of mixing. Depending on the capillary number N_{Ca}, where

$$N_{Ca} = \frac{\rho_L \nu_L U_b}{\sigma}, \tag{5.11}$$

either counter-rotating flow inside the liquid slug or a complete bypass of liquid around the bubble were observed, in agreement with Taylor. For counter-rotating flow, they developed a theory for average recirculation time within the slug vortices. They defined a dimensionless recirculation time τ_{cir}, as the time for the liquid to move from one end of the slug to the other divided by a reference time. The reference time was the slug length l_s divided by the bubble velocity U_b, or l_s/U_b.

Imposing laminar conditions and using analysis from an earlier paper,[28] Thulasidas *et al.*[25] derived an expression for the dimensionless recirculation time,

$$\tau_{cir} = \frac{\psi}{1-\psi/2}, \tag{5.12}$$

where $\psi = U_b/U_m$. With the help of previous work,[28,29] Liu *et al.*[30] proposed an empirical correlation for N_{Ca},

$$N_{Ca} = 4.472\frac{(\psi-1)^3}{\psi^2}. \tag{5.13}$$

This correlation allows $\psi = U_b/U_m$ to be calculated from the capillary number, and the dimensionless recirculation time to be calculated from eqn (5.12).

For the dimensions and physical properties of the slug-flow crystallization unit of Jiang *et al.*,[2] the calculated dimensionless recirculation time *versus* N_{Ca} behavior is shown in the top curve in Figure 5.9, and $\psi = U_b/U_m$ is also plotted as the bottom curve. The plots in Figure 5.9 are confined to Reynolds numbers less than $Re_L = 2100$, as laminar conditions must hold not only for eqn (5.12) to apply, but also for the desired range of slug flow to exist.