$$\frac{\partial n(L, G_{\mathrm{R}}, t)}{\partial t} + G\frac{\partial\left[n(L, G_{\mathrm{R}}, t)\right]}{\partial L} = B\delta(L - L_n) + \frac{n_f(L, G_{\mathrm{R}}, t) - n(L, G_{\mathrm{R}}, t)}{\tau} \tag{2.83}$$

Subject to corresponding boundary condition:

$$\lim_{\substack{L\to\infty \\ G_{\mathrm{R}}\to\infty}} n(L, G_{\mathrm{R}}, t) = 0 \tag{2.84}$$

The energy and mass balance equations for brevity are not repeated here. It was shown in the literature that it is possible to fit the parameters of GRD functions, listed in Table 2.14, to the steady-state CSD collected in MSMPR crystallizers.[48] In the same work it was concluded that there is only a slight difference between the different probability density function performances in the fitting of the experimental data.

The RF model assumes constant growth rate with random fluctuations around the growth rate, noted by F_{g}, which can be modeled as:

$$F_{\mathrm{g}} = \frac{\partial\left[D_{\mathrm{g}} n(L, t)\right]}{\partial L} \tag{2.85}$$

where D_{g} stands for the growth rate diffusivity. The PBE with nucleation, growth and RF mechanisms for an MSMPR crystallizer can be written as:

$$\frac{\partial n(L,t)}{\partial t} + \frac{\partial\left[\mathrm{Gn}(L,t)\right]}{\partial L} - \frac{\partial^2\left[D_{\mathrm{g}} n(L,t)\right]}{\partial L^2} = B\delta(L - L_n) + \frac{n_f(L,t) - n(L,t)}{\tau} \tag{2.86}$$

As it can be seen, RF gives rise to a second order term in the PBE, which is a diffusive term in the partial differential equation. It must be noted that from a theoretical perspective, RF and CCG approaches are not exclusive, but are rarely combined with each other.

Appendix

A1 Derivation of the Population Balance Equation for Plug Flow Crystallizer

The general balance equation for a quantity in a control volume can be written as follows.

$$\begin{bmatrix}\text{Rate of}\\ \text{accumulation}\end{bmatrix} = \begin{bmatrix}\text{Rate of}\\ \text{entry}\end{bmatrix} - \begin{bmatrix}\text{Rate of}\\ \text{departure}\end{bmatrix} \pm \begin{bmatrix}\text{Rate of}\\ \text{generation/ depletion}\end{bmatrix}$$

We consider the number density function n for the crystals in a PFC which depends on one internal coordinate size L, one external coordinate – axial location along the crystallizer x and time t. Taking number balance at the volume element $A\Delta x\Delta L$