equilibrium batch process or even greater.[46] However, the increase of yield by using recycle stream can be limited if there is a build-up of impurities within the system.[46]

2.7 The Growth Rate Dispersion (GRD)

In the traditional crystallization modeling practice, the McCabe's ΔL law is applied, which states that the growth rate of crystals is constant in time and it doesn't depend on the crystal size. An important consequence of this law is that two crystals under identical thermodynamic conditions (temperature, supersaturation *etc.*) exhibit identical growth rate. If this is true, in a seeded batch pure growth crystallization process the width of the CSD should remain constant throughout the growth process. This is a reasonable approximation of growth rate in numerous cases, but for many systems strong deviations were observed from the McCabe's ΔL law. This experimental evidence suggests that some microscopic mechanisms must exist that can cause a fluctuation of the observed growth rate, therefore, there is a growth rate variance in a population of crystals under macroscopically identical conditions.

It seems reasonable to assume that GRD is caused by the interplay of various mechanisms, whose relative importance depends on the crystalline system, the operating conditions and the particle history. From a modeling perspective, there are two general techniques to describe GRD: the random fluctuations (RF) and constant crystal growth (CCG) approach. The CCG assumes that each crystal is intrinsically born with an inherent growth rate, which can differ from crystal to crystal, but it remains constant over the lifetime of the crystal. Practically, in the CCG model the growth rate is often assumed to be some standard probability density function (normal, lognormal, gamma *etc.*). In contrast, RF assumes that the growth rate of crystals fluctuates in time around a given, constant expected value. The spread of growth rates is described by a coefficient of dispersion, that gives rise to a second order differential.

For the illustration of CCG, let $F(G_\mathrm{R})\mathrm{d}G_\mathrm{R}$ represent the distribution of relative growth rates around a nominal value[1] for the crystals such that $F(G_\mathrm{R})\mathrm{d}G_\mathrm{R}$ is the fraction of the total number of crystals having a relative growth rate of G_R. Table 2.14 gives the typically applied probability density functions for approximating the growth rate dispersion (Figure 2.17).[48,49]

Then, for the characterization of the crystal population, which also takes into account the growth rate of individual crystals, a bivariate population density function is introduced $n(L,G_\mathrm{R},t)$, which gives the number of crystals in the $(L,L + dL) \times (G_\mathrm{R},G_\mathrm{R} + dG_\mathrm{R})$ size and relative growth rate domain in t time moment in unit volume of suspension. The size density function can be extracted from the bivariate population density function, since the joint probability density function is related to the normalized density function, by the integral: