for case C:

$$V = \frac{\pi R^3}{3}\left(1-\cos\theta\right)^2\left(2+\cos\theta\right)+\pi R^2 h\sin^2\theta-\frac{\pi}{3}ms^2h \tag{8.10a}$$

$$A_L = 2\pi R^2(1-\cos\theta) \tag{8.10b}$$

$$A_{SL} = \pi R^2\sin^2\theta - m\pi s^2 + ms\sqrt{s^2+h^2} \tag{8.10c}$$

where m is the number of asperities on the surface, h (m) is the profile height, l (m) is the distance between two consecutive peeks, s (m) is the half-base length of an asperity.

After eqn (8.4a–c) and eqn (8.10a–c):

$$\begin{aligned}\Delta G = &-\frac{\pi R^3}{3}\frac{\Delta\mu}{\Omega}\left(1-\cos\theta\right)^2\left(2+\cos\theta\right)\\ &+2\pi R^2\gamma_L\left(1-\cos\theta\right)-\pi R^2\left(\gamma_s-\gamma_i\right)\sin^2\theta\end{aligned} \tag{8.11}$$

To progress further, a mathematical correlation is required to link γ_L and the term $(\gamma_s - \gamma_i)$. When nucleation occurs on a non-porous (dense) surface, the Young equation can be used to express the mechanical equilibrium between interfaces:

$$(\gamma_s - \gamma_i) = \gamma_L\cos\theta \tag{8.12}$$

For a porous membrane, a modified form of the Young equation correlating the surface porosity to the measured and equilibrium contact angles can be adopted:[23]

$$\left(\gamma_s-\gamma_i\right)=\gamma_L\left[\cos\theta+\frac{4\varepsilon\left(1+\cos\theta\right)}{(1-\varepsilon)\left(1-\cos\theta\right)}\right] \tag{8.13}$$

For a rough membrane surface, the wetting behavior can be adequately described by the Wenzel equation:

$$\left(\gamma_s-\gamma_i\right)=\frac{1}{r_W}\gamma_L\cos\theta \tag{8.14}$$

where r_W is the Wenzel roughness factor ($r_W > 1$). For the case illustrated in Figure 8.3c, when assuming that asperities on the surface have a conical shape, the roughness factor r can be evaluated as:

$$r_W = \frac{\left(\pi l^2 - m\pi s^2\right)+m\pi s\sqrt{h^2+s^2}}{\pi l^2} \tag{8.15}$$

In order to determine the Gibbs free energy barrier ΔG^* reached for a critical nucleus having radius R^*, the following maximization condition has to be applied: