difference in dependency on supersaturation between crystal growth and nucleation inherently leads to strong nonlinear process behaviour (*e.g.*, oscillatory behaviour) and a steady state corresponding to a desired CSD may not be stable,[93] which motivates the use of feedback control to stabilize the process operation and obtain the desired properties of the CSD. As discussed earlier, a typical control loop would use some property of the CSD as measured and controlled variable and would use the flow rate through the fines dissolution loop as manipulated variable. An alternative control structure could use the inlet solute concentration as manipulated variable.

The first step in the framework from Christofides and co-workers is to apply a model reduction step. The aim is to capture the dominant dynamics of the process described by a limited number of ordinary differential equations (ODEs) so that practical nonlinear model-based feedback controllers can be designed. A general model reduction approach based on the method of the weighted residuals and the concept of inertial manifold is provided by Chiu and Christofides.[93] The method coincides with the well-known method of moments when the basis functions to expand the CSD are Laguerre polynomials and the weighting functions have a specific form. The *i*th moment of the CSD is defined as follows:

$$m_i = \int_0^\infty L^i n(L,t)\,\mathrm{d}L,\ i = 0,1,2,\ldots, \tag{4.6}$$

multiplying the population balance in eqn (4.3) with L^i, substituting (4.4), assuming spherical particles (*i.e.*, $k_\mathrm{v} = 4/3\pi$), assuming growth with a first-order dependency on supersaturation, and using the following kinetic relation for crystal nucleation,

$$B = \varepsilon k_\mathrm{A} \mathrm{e}^{\frac{-k_\mathrm{B}}{\left[(C/C_\mathrm{s})-1\right]^2}}, \tag{4.7}$$

and the following (sharp) classification function:

$$h(L) = \begin{cases} 1, \text{ if } L \le L_\mathrm{m}, \\ 0, \text{ if } L > L_\mathrm{m}, \end{cases} \tag{4.8}$$

and, finally, integrating over all crystal sizes gives the following set of ODEs:[94]

$$\begin{aligned} \frac{\mathrm{d}m_0}{\mathrm{d}t} &= -\frac{m_0}{\tau} + \left(1 - \frac{4}{3}\pi m_3\right) k_\mathrm{A} \mathrm{e}^{\frac{-k_\mathrm{B}}{\left[(C/C_\mathrm{S})-1\right]^2}} - \int_0^{L_\mathrm{m}} \frac{n(L,t)}{\overline{\tau}}\mathrm{d}L, \\ \frac{\mathrm{d}m_i}{\mathrm{d}t} &= -\frac{m_i}{\tau} + \mathrm{i}k_\mathrm{G}(c - c_\mathrm{S})m_{i-1} - \int_0^{L_\mathrm{m}} L^i \frac{n(L,t)}{\overline{\tau}}\mathrm{d}L,\ i = 1,2,\ldots, \\ \frac{\mathrm{d}c}{\mathrm{d}t} &= \frac{c_0 - c - 4\pi\tau(c - c_\mathrm{S})m_2(\rho - c)}{\tau\left(1 - \frac{4}{3}\pi m_3\right)}. \end{aligned} \tag{4.9}$$