$$\mu_k = \int_0^\infty L^k n(L,t)\,\mathrm{d}L,\ k = 0,1,2\ldots \tag{2.46}$$

where μ_k (μm^k) denotes the k-th moment of the distribution. The first four moments (k = 0, 1, 2, 3) have physical meaning, being proportional, in row, with the specific number, length, surface area and volume of particles. Superior moments can also be computed, but no physical meaning can be associated to them.

Applying the model transformation rule eqn (2.46) on the PB eqn (2.31).

$$\begin{aligned}&\int_0^\infty L^k \frac{\partial n(L,t)}{\partial t}\mathrm{d}L + \int_0^\infty L^k \frac{\partial\left[Gn(L,t)\right]}{\partial L}\mathrm{d}L\\ &= \int_0^\infty L^k B\delta(L-L_n)\mathrm{d}L + \int_0^\infty L^k B_{\mathrm{agg}}\,\mathrm{d}L - \int_0^\infty L^k D_{\mathrm{agg}}\,\mathrm{d}L\\ &+\int_0^\infty L^k B_{\mathrm{bre}}\,\mathrm{d}L - \int_0^\infty L^k D_{\mathrm{bre}}\,\mathrm{d}L + \int_0^\infty L^k \frac{n_f(L,t)-n(L,t)}{\tau}\mathrm{d}L\end{aligned} \tag{2.47}$$

Depending on the form of kinetic functions applied in eqn (2.47), discussed in the first section of this chapter, the integrals might or might not be evaluated,[4] which delimits the applicability of the SMOM for numerous practical problems. For nucleation and growth processes eqn (2.47) simplifies to:

$$\begin{aligned}\frac{\mathrm{d}\mu_k}{\mathrm{d}t} &= \int_0^\infty L^k B\delta(L-L_n)\mathrm{d}L - \int_0^\infty L^k \frac{\partial\left[Gn(L,t)\right]}{\partial L}\mathrm{d}L\\ &+\int_0^\infty L^k \frac{n_f(L,t)-n(L,t)}{\tau}\mathrm{d}L\end{aligned} \tag{2.48}$$

Which, assuming size independent growth, applying the moment transformation rule eqn (2.46) leads to the system of moment equations:

$$\frac{\mathrm{d}\mu_k}{\mathrm{d}t} = BL_n{}^k + kG\mu_{k-1} + \frac{\mu_{f,k}-\mu_k}{\tau} \tag{2.49}$$

Eqn (2.49) is an ordinary differential equation, which can be solved simultaneously with the mass and energy balance equations. Although they are averaged quantities, the moments enable the calculation of numerous relevant statistical properties of the particle population, as summarized in Table 2.9.

Table 2.9 gives various mean sizes and distributional properties, but the exact reconstruction of the CSD is not possible. However, various methods have been developed to approximate the original CSD based on its leading moments.[3]

Even though the integrals of eqn (2.47) can be evaluated, the resulting equations system might not be closed. A typical example is the size dependent growth:

$$G(L) = GL^j \tag{2.50}$$