Grashof number $\mathrm{Gr} = \frac{D^3 \rho^2 g \beta \Delta T}{\mu^2}$ (g: gravity acceleration, β: thermal expansion coefficient)

Prandtl number $\mathrm{Pr} = \frac{c_\mathrm{p} \mu}{k}$ (c_p: specific heat)

Graetz number (for hollow fiber membrane modules with length L and mass flowrate $\dot{m}$): $\mathrm{Gr} = \frac{\dot{m} c_\mathrm{p}}{kL}$

Since the membrane acts as hetero-nucleant interface with low surface, and due to concentration polarization that increases the solute concentration in proximity of the membrane, the nucleation process in a membrane crystallizer preferentially occurs on the membrane module; crystals are then flushed and recirculated into a separate crystallizer tank by the retentate flow. The combination of membrane modules and temperature-controlled crystallization should be considered as one system consisting of both nucleation and growth steps. Moreover, the retentate flowrate plays a crucial role in determining the detachment of crystals from the membrane surface and, in general, on the overall crystallization process.

For a continuous process, the growth rate (G) is estimated from the variation of the overall crystal length ($\overline{\mathrm{L}}$) as a function of the residence time (τ):

$$G = \frac{\mathrm{d}\bar{L}(\tau)}{\mathrm{d}\tau} \approx \frac{\bar{L}(\tau_{n+1}) - \bar{L}(\tau_n)}{(\tau_{n+1} - \tau_n)} \tag{8.28}$$

where:

$$\bar{L}(\tau) \approx \sum_i \alpha_i(\tau) L_i \tag{8.29a}$$

$$\alpha_i(\tau) = \frac{N_i(\tau)}{N_\mathrm{total}(\tau)} \tag{8.29b}$$

The crystal growth rate is empirically related to the supersaturation (S):

$$G = k_\mathrm{g} S^g \tag{8.30}$$

where k_g is the growth rate constant (function of temperature, fluid-dynamics, presence of impurity *etc.*) and exponent g represents the kinetic order of the growth process. For NaCl crystallization on microporous polypropylene membranes carried out at low supersaturation ratio (S = 0.065–0.097), least square multiple linear regression analysis of data resulted in $g = 1.17$ and $k_\mathrm{g} = 1.92 \times 10^{-5}$ (G expressed in m s^{-1}).[2]