



**Fig. 3** Design space graph for primary drying stage of lyophilization with design space, control space, and set point

where  $\dot{Q}$  is the heat transfer rate (cal/s),  $A_v$  is the outer cross-sectional area of the vial ( $\text{cm}^2$ ),  $K_v$  is the vial heat transfer coefficient (cal/s. $\text{cm}^2$ .K),  $T_s$  is the shelf surface temperature ( $^{\circ}\text{C}$ ), and  $T_p$  is the product temperature at the sublimation interface ( $^{\circ}\text{C}$ ).

Mass transfer of water vapor through the dried product layer (ignoring the small resistance of the stopper) can be represented using [25]:

$$\frac{dm}{dt} = A_p \cdot \frac{P_{\text{ice}} - P_c}{\hat{R}p} \tag{2}$$

where  $dm/dt$  is the sublimation rate per vial (g/h/vial),  $A_p$  is the internal cross-sectional area of the vial ( $\text{cm}^2$ ),  $P_{\text{ice}}$  is the vapor pressure of ice (Torr),  $P_c$  is the chamber pressure (Torr), and  $\hat{Q}$  is the area-normalized dry layer resistance ( $\text{cm}^2$ .Torr.h.g $^{-1}$ ) of the product in the vial.  $P_{\text{ice}}$  (the vapor pressure of water above ice at temperature  $T_{\text{ice}}$ ) can be expressed as [26]:

$$P_{\text{ice}} = 2.69 \times 10^{13} \times e^{-6144.96/T_{\text{ice}}} \tag{3}$$

where  $T_{\text{ice}}$  (or  $T_p$ ) is the temperature of the ice (or product) at the sublimation interface (K) which can be expressed as [26]:

$$T_p = T_b \frac{(\dot{Q}) \cdot L'}{A_v \cdot K} \tag{4}$$