3.3.2.1 Measure of Closeness Based on the Absolute Difference

It should be noted that we have

$$(y-\hat{y}) \sim N\left(0,(1+c)\sigma_e^2\right).$$

Therefore, p_1 can be estimated by

$$\hat{p}_1 = \Phi\left(\frac{\delta}{\sqrt{(1+c)\hat{\sigma}_e^2}}\right) - \Phi\left(\frac{-\delta}{\sqrt{(1+c)\hat{\sigma}_e^2}}\right).$$

Using the delta method through a Taylor expansion, for a sufficiently large sample size n,

$$\text{var}\left(\hat{p}_1\right) \approx \left(\phi\left(\frac{\delta}{\sqrt{(1+c)\sigma_e^2}}\right) - \phi\left(\frac{-\delta}{\sqrt{(1+c)\sigma_e^2}}\right)\right)^2 \frac{\delta}{2(1-\delta)(n-2)\sigma_e^2},$$

where $\phi(z)$ is the probability density function of a standard normal distribution. Furthermore, $\text{var}\left(\hat{p}_1\right)$ can be estimated by V_1, where V_1 is given by

$$V_1 = \frac{2\delta^2}{(1+c)(n-2)\hat{\sigma}_e^2}\phi^2\left(\frac{\delta}{\sqrt{(1+c)\hat{\sigma}_e^2}}\right).$$

By Slutsky's theorem, $\hat{p}_1 - p_0 / \sqrt{V_1}$ can be approximated by a standard normal distribution. For the testing of the hypotheses H_0: $p_1 \leq p_0$ versus H_a: $p_1 > p_0$, we would reject the null hypothesis H_0 if

$$\frac{\hat{p}_1 - p_0}{\sqrt{V_1}} > z_{1-\alpha},$$

where $z_{1-\alpha}$ is the $100(1-\alpha)$th percentile of a standard normal distribution.

3.3.2.2 Measure of Closeness Based on the Relative Difference

In other words, for evaluation of p_2, we note that y^2 and $\hat{y}^2$ follow a noncentral χ_1^2 distribution with noncentrality parameter μ_0^2/σ_e^2 and $\mu_0^2/c\sigma_e^2$, respectively, where $\mu_0 = \hat{\beta}_0 + \hat{\beta}_1 x$. Hence, $c\hat{y}^2/y^2$ is doubly noncentral F distributed with $\upsilon_1 = 1$ and $\upsilon_2 = 1$ degrees of freedom and noncentrality parameters $\lambda_1 = \mu_0^2/c\sigma_e^2$ and $\lambda_2 = \mu_0^2/\sigma_e^2$. According to Johnson and Kotz (1970), a noncentral F distribution can be approximated by

$$\frac{1+\lambda_1\upsilon_1^{-1}}{1+\lambda_2\upsilon_2^{-1}}F_{\upsilon,\upsilon'}$$