the dry network dimensions, in Equation 19 the undeformed but swollen network is the reference state. The impact of swelling on the mechanical properties of a swollen gel is highlighted by the $1/\lambda_s$ factor in Equation 19. While the modulus of the dry network is $G_o = k_B T\upsilon / V_o$, the swollen network has a reduced modulus of $G = k_B T\upsilon/\lambda_s V_o$. As λ_s is related to the swelling ratio of the network by $\lambda_s = q^{1/3}$, the two moduli are related to each other through the swelling ratio:

$$G = G_o/q^{1/3} \tag{20}$$

The physical meaning of Equation 20 is that swelling only has a diluting effect on the stiffness of the network. Note that the swelling ratio in Equation 20 does not need to be the equilibrium swelling ratio, Q_{eq}. The swelling ratio will reach its equilibrium value when the overall free energy is at a minimum with respect to any change in the number of solvent molecules within the swollen network:

$$\partial \Delta F/\partial n_s = 0 \tag{21a}$$

Equation 21a can be expanded using Equation 13 to obtain:

$$\partial \Delta F_{el}/\partial n_s + \partial \Delta F_m/\partial n_s = 0 \tag{21b}$$

Recalling any of the statistical models presented in the previous section, the elastic free energy term in Equation 21b can be determined as a function of network deformation. When there is no external load applied, the extension ratios resulted from the equilibrium swelling of the network is $\lambda_{s,o}$ which can be related to the free equilibrium swelling ratio of the network, $Q_{eq,o}$ by:

$$\lambda_{s,o} = (Q_{eq,o})^{1/3} \tag{22}$$

For an affine network, the elastic free energy as a function of equilibrium swelling extension ratio is:

$$\Delta F_{el} = (3V_o/2)(\rho RT/M_c)(\lambda_{s,o}^2 - 1) \tag{23}$$

Separately, for any swelling ratio the following relationship exists between mole number of solvent molecules per unit volume of dry network, n_s/V_o, and the molar volume of the solvent molecules, v_s:

$$q = 1 + (n_s/V_o)v_s \tag{24}$$

Equation 24 guarantees the molecular incompressibility of the system and is independent of the hypothesis presented in the form of Equation 13. At equilibrium under free swelling conditions $q \rightarrow Q_{eq,o}$ in Equation 24. Now, by inserting Equation 22 in Equation 23, and using Equation 24 to establish a correlation between $Q_{eq,o}$ and n_s, the equilibrium swelling ratio can be determined from Equation 21b and Equation 14 as:

$$\text{In}\left(1 - \frac{1}{Q_{eq,o}}\right) + \frac{1}{Q_{eq,o}} + \frac{\chi}{Q_{eq,o^2}} + \frac{(\rho v_s / M_c)}{Q_{eq,o^{1/3}}} = 0 \tag{25}$$

Solving Equation 25 for any given χ, v_s and ρ/M_c will give the equilibrium swelling ratio when no external stress is applied. For hydrogels swollen in water v_s is