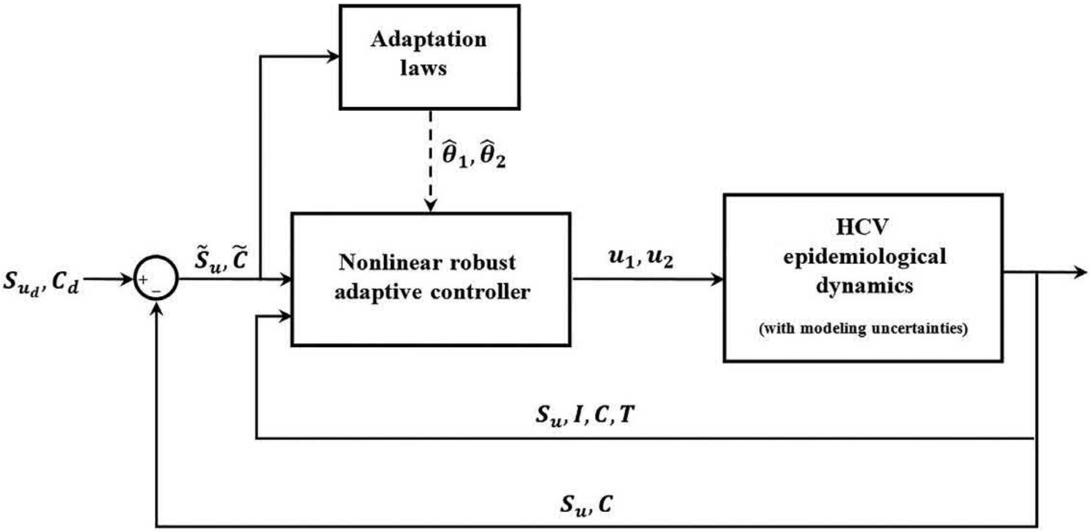

FIG. 2 Conceptual diagram of the nonlinear adaptive method developed to control the HCV epidemic in the existence of uncertainties on the model parameters.

In the following section, taking advantage of the Lyapunov stability theorem, it will be proven that the control laws (11), (12) together with some adaptation laws guarantee the tracking convergence, stability, and robustness for the treatment of HCV outbreak.

4.2. Stability Proof and Adaptation Laws

The closed-loop dynamics of the system is achieved first by substituting the control laws (11), (12) into the dynamics of the HCV epidemic (1):

$$\frac{\dot{\tilde{S}}_u + \lambda_1 \tilde{S}_u}{S_u} = -Y_1 \tilde{\theta}_1 \quad (13)$$

$$\frac{\dot{\tilde{C}} + \lambda_2 \tilde{C}}{C} = -Y_2 \tilde{\theta}_2 \quad (14)$$

where $\tilde{\theta}_i$ (for $i = 1, 2$) is defined as $\hat{\theta}_i - \theta_i$.

The adaptation laws are designed to update parameters' estimation to keep the system's robustness against uncertainties, as

$$\dot{\hat{\theta}}_1^T = S_u \tilde{S}_u Y_1 \Gamma_1 \quad (15)$$

$$\dot{\hat{\theta}}_2^T = C \tilde{C} Y_2 \Gamma_2 \quad (16)$$

where Γ_1 and Γ_2 are the adaptation gain matrices and considered to be positive definite.

Now, employing the Lyapunov stability theorem [27] and based on the previously derived closed-loop dynamics (13), (14) and the designed adaptation laws (15) and (16), the tracking convergence, stability and robustness for the aware susceptible and chronically infected classes will be proven. With this aim, a positive definite Lyapunov candidate function is selected as

$$V = \frac{1}{2}\left[\tilde{S}_u^2 + \tilde{C}^2 + \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 + \tilde{\theta}_2^T \Gamma_2^{-1} \tilde{\theta}_2\right] \quad (17)$$

The Lyapunov function's time derivative is determined:

$$\dot{V} = \tilde{S}_u \dot{\tilde{S}}_u + \tilde{C} \dot{\tilde{C}} + \dot{\tilde{\theta}}_1^T \Gamma_1^{-1} \tilde{\theta}_1 + \dot{\tilde{\theta}}_2^T \Gamma_2^{-1} \tilde{\theta}_2 \quad (18)$$

It should be mentioned that $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$ because θ is constant ($\dot{\theta}$ is zero). By substituting the adaptation laws (15) and (16) into Eq. (18), the time derivative of V is simplified to

$$\dot{V} = -\lambda_1 \tilde{S}_u^2 - \lambda_2 \tilde{C}^2 \quad (19)$$

As mentioned in the previous descriptions, λ_1 and λ_2 are considered to be positive; thus, the Lyapunov function's time derivative is negative semidefinite. Thus, based on Barbalat's lemma (described in Appendix) and the Lyapunov stability theorem [27], it is proven that $\tilde{S}_u$ and $\tilde{C}$ converge to the zero. In other words, employing the suggested adaptive feedback control strategy ensures the tracking convergence and stability ($\tilde{S}_u \rightarrow 0$ and $\tilde{C} \rightarrow 0$ as $t \rightarrow \infty$) in the presence of uncertainties. Thus, the numbers of unaware susceptible (S_u) and chronically infected (C) people converge to the desired values ($S_u \rightarrow S_{u_d}$ and $C \rightarrow C_d$).