



FIG. 11 The difference between (A) unaware susceptible population and its desired value ($\tilde{S}_u = S_u - S_{ud}$) and (B) chronically infected population and its desired value ($C = C - C_d$) for different parametric uncertainty levels.

APPENDIX. BARBALAT'S LEMMA

The Lyapunov function $V(\tilde{S}_u, \tilde{C}, \tilde{\theta}_1, \tilde{\theta}_2)$ in Eq. (17) is positive definite and its time derivative ($\dot{V}(\tilde{S}_u, \tilde{C})$) in Eq. (19) is negative semidefinite. Thus, based on the Lyapunov stability theorem [27], V is bounded and it is concluded that $S_u, \tilde{C}, \tilde{\theta}_1$, and $\tilde{\theta}_2$ remain bounded.

Barbalat's lemma: If g is a uniformly continuous function and $\lim_{t \rightarrow \infty} \int_0^t g(\eta) d\eta$ exists and has a finite value, it is guaranteed that [27]

$$\lim_{t \rightarrow \infty} g(t) = 0 \tag{A.1}$$

In order to use this lemma for the controlled system of HCV outbreak, $g(t)$ is considered to be $-\dot{V}$:

$$g(t) = -\dot{V} = \lambda_1 \tilde{S}_u^2 + \lambda_2 \tilde{C}^2 \tag{A.2}$$

By integrating both sides of Eq. (A.2), one can write:

$$V(0) - V(\infty) = \lim_{t \rightarrow \infty} \int_0^t g(\eta) d\eta \tag{A.3}$$

Since \dot{V} is negative, $V(0)$ is larger than $V(\infty)$ and $V(0) - V(\infty) \geq 0$. Moreover, as mentioned previously, V is bounded based on the Lyapunov stability theorem. Thus, $\lim_{t \rightarrow \infty} \int_0^t g(\eta) d\eta$ in Eq. (A.3) exists and has a bounded value. Therefore, it is concluded using the Barbalat's lemma that

$$\lim_{t \rightarrow \infty} (\lambda_1 \tilde{S}_u^2 + \lambda_2 \tilde{C}^2) = 0 \tag{A.4}$$

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