TABLE 1
Parameters of the Mathematical Model of the HCV (1) [4].

Parameter	Description
b	Rate of birth
μ	Rate of death
β	Transmission coefficient
K_1	Chronic stage infectiousness relative to acute stage
K_2	Treated individuals' infectiousness relative to acute ones
α	Rate of being infected for aware people relative to unaware ones
γ	Leaving rate of acutely infected class
q	Progressing proportion from acute stage to chronic one
ξ	Transferring rate from treated class to other ones
p	Moving back proportion from treated class to chronic one
θ	HCV-induced death rate

4. NONLINEAR ADAPTIVE CONTROLLER FORMULATION FOR EPIDEMIOLOGY OF HCV

In this section, a new nonlinear adaptive controller is formulated for the uncertain hepatitis C virus epidemic. The main purpose of the control method is to minimize the populations of unaware susceptible (S_u) and chronically infected (C) classes. Two control inputs $u_1(t)$ and $u_2(t)$ are considered in order to reach this objective. $u_1(t)$ denotes the effort rate to inform the susceptible individuals from the HCV by media publicity, educational campaigns, public service advertising, and so on, and $u_2(t)$ is employed to reflect the rate of treatment on chronically infected individuals [4].

Using the above-mentioned control inputs, the populations of unaware susceptible (S_u) and chronically infected (C) classes will decrease by tracking some desired values. Moreover, due to the decrease of the mentioned components, the number of aware susceptible (S_a) and treated (T) individuals will increase and decrease, respectively. The Lyapunov theorem is employed to prove the stability of the closed-loop system. In addition, some adaptation laws are defined in order to update the estimated parameters of the system to guarantee the stability and robustness of the system against the uncertainties of the dynamic model. A conceptual diagram of the proposed nonlinear feedback controller with the adaptive scheme is illustrated in Fig. 2.

4.1. Nonlinear Adaptive Control Laws

Control inputs ($u_1(t)$, $u_2(t)$) could be calculated using dynamics of the unaware susceptible and chronically infected compartments from Eq. (1) as

$$u_1 = -\frac{\dot{S}_u}{S_u} + \frac{b}{S_u} - \frac{\beta}{N}(I + K_1 C + K_2 T) - \mu + (1-q)\gamma \frac{I}{S_u} \quad (3)$$

$$u_2 = -\frac{\dot{C}}{C} + q\gamma\frac{I}{C} - (\mu + \theta) + p\xi\frac{T}{C} \quad (4)$$

Property. *The right-hand sides of Eqs. (3), (4) can be linearly parameterized in terms of their available parameters.* ϕ_1 *and* ϕ_2 *are considered to be the arbitrary variables instead of* $\dot{S}_u$ *and* $\dot{C}$.

Now, the right sides of the above equations can be rewritten as

$$-\frac{\dot{S}_u}{S_u} + \frac{b}{S_u} - \frac{\beta}{N}(I + K_1 C + K_2 T) - \mu + (1-q)\gamma \frac{I}{S_u} = -\frac{\phi_1}{S_u} + Y_1\theta_1 \quad (5)$$

$$-\frac{\dot{C}}{C} + q\gamma\frac{I}{C} - (\mu + \theta) + p\xi\frac{T}{C} = -\frac{\phi_2}{C} + Y_2\theta_2 \quad (6)$$

where Y_1 *and* Y_2 *are the regressor matrices, contain known functions of HCV epidemic variables.* θ_1 *and* θ_2 *are the parameter vectors, which contain unknown parameters of the dynamic (Eqs. 7, 8). Accordingly, these matrices and vectors are defined as*

$$Y_1 = \begin{bmatrix} \frac{1}{S_u} & -\frac{I}{N} & -\frac{C}{N} & -\frac{T}{N} & \frac{I}{S_u} & -1 \end{bmatrix}; \quad \theta_1 = [b \;\; \beta \;\; \beta K_1 \;\; \beta K_2 \;\; (1-q)\gamma \;\; \mu]^T \quad (7)$$

$$Y_2 = \begin{bmatrix} \frac{I}{C} & \frac{T}{C} & -1 \end{bmatrix}; \quad \theta_2 = [q\gamma \;\; p\xi \;\; (\mu+\theta)]^T \quad (8)$$

This regressor presentation is used to summarize the equations and define the adaptation and control laws. In order to design nonlinear control laws, two new variables ϕ_1 and ϕ_2 are defined as follows:

$$\phi_1 = \dot{S}_{u_d} - \lambda_1 \tilde{S}_u \quad (9)$$

$$\phi_2 = \dot{C}_d - \lambda_2 \tilde{C} \quad (10)$$

where λ_1 and λ_2 are the controller gains and considered to be positive and constant. Now, the nonlinear adaptive control laws are defined as

$$u_1 = -\frac{\dot{S}_{u_d} - \lambda_1 \tilde{S}_u}{S_u} + Y_1\hat{\theta}_1 \quad (11)$$

$$u_2 = -\frac{\dot{C}_d - \lambda_2 \tilde{C}}{C} + Y_2\hat{\theta}_2 \quad (12)$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the vectors of estimated parameters.