The amount absorbed after infinite time is the amount in the skin before the vehicle is removed, which is equal to M_∞.

Given the complexity of solute–distance–time profiles in membranes, the expressions for these profiles are not reproduced here. However, it should be emphasized that these may be important, as illustrated in the use of in vivo attenuated total reflection–Fourier transform infrared spectroscopy (ATR–FTIR) to examine the kinetics of solute uptake into human SC in vivo [7].

2.1.3 In Vitro Permeability Studies with a Constant Donor Concentration and Finite Receptor Volume

In most in vitro studies, it is assumed that sink conditions apply in the receptor phase. However, the receptor phase is a finite volume, and solute accumulation may be possible if there is an inadequate removal rate of the solute penetrating through. Siddiqui et al. [8] related the steady-state flux of steroids through human epidermis to the differences in concentrations between donor C_v and receptor C_{ss} concentrations. In the present notation, this equation is:

$$J_{ss} = k_p\left(C_v - \frac{C_{ss}}{K_r}\right) \tag{2.28}$$

where K_r is the partition coefficient between the membrane and vehicle $K_r = C_r/C_v$ and K_m is the partition coefficient between the membrane and vehicle $K_m = C_m/C_v$. Siddiqui et al. [8] assumed that $K_r = 1$.

Implicit in the underlying boundary conditions for the receptor phase is a constant clearance of solute Cl_r due to repeated sampling or use of a flow-through cell. If such a clearance were absent, C_{ss} would continually increase and approach C_vK_r. The value of C_{ss} is defined by the relative magnitudes of the clearance Cl_r and k_pA/K_r:

$$C_{ss} = \frac{k_pAC_v}{Cl_r + \left(k_pA/K_r\right)} \tag{2.29}$$

Siddiqui et al. [8] also applied this equation and the dermal clearance of solutes Cl_r to predict the steady-state epidermal concentrations of solutes C_{ss}. Roberts [9] considered the limits of large k_pA as it exists for phenol absorption and low k_pA as it exists for steroid absorption. He suggested that when $k_pA \gg K_rCl_r$, C_{ss} would eventually approach the donor concentrations used (C_vK_r). In contrast, when $k_pA \ll K_rCl_r$, C_{ss} approaches k_pAC_v/Cl_r.

The derivation of the full equation, from which steady-state Equations (2.28) and (2.29) arise, needs to take into account a finite receptor or epidermis volume. The boundary condition at $x = h_m$ in this case is $C_m(h_m, t)/K_m = C_r(t)/K_r$, together with [10]:

$$V_r\frac{dC_r}{dt} = -AD_m\frac{\partial C_m}{\partial x}\Big|_{x=h_m} - Cl_rC_r \tag{2.30}$$

where Cl_r is the clearance (mL/min) of solution containing solute from the receptor phase, V_r is the volume of the receptor, and C_r is the concentration in the receptor.

Using this boundary condition together with the boundary condition in [7] yields the amount of solute, which penetrated the skin into the receptor (= amount in receptor + amount cleared from receptor), and for the flux of solute into the receptor [9]:

$$\hat{Q}(s) = \frac{k_pAC_v}{s^2}\frac{\sqrt{st_d}}{\sinh\sqrt{st_d} + \left\{\left[\sqrt{st_d}/\left(st_dV_{rN} + Cl_{rN}\right)\right]\cosh\sqrt{st_d}\right\}} \tag{2.31}$$

$$\hat{J}_s(s) = \frac{k_pC_v}{s}\frac{\sqrt{st_d}}{\sinh\sqrt{st_d} + \left\{\left[\sqrt{st_d}/\left(st_dV_{rN} + Cl_{rN}\right)\right]\cosh\sqrt{st_d}\right\}} \tag{2.32}$$